

PROJECTIONS OF TRIADIC PREDICATES;

! This chapter speaks briefly of projections of triadic predicates. If  $T$  is a triadic predicate, then  $(T \diamond c)$  is its projection by  $c$  (in its second argument). That is,  $x$  bears  $(T \diamond c)$  to  $y$  if and only if the triple  $x, c$ , and  $y$  satisfy  $T$ .

Other than the Fundamental Proposition, the only proposition asserted is about the inverse of a projection (P5). i

! 1.  $\diamond$  represents projection (on a triadic predicate). i

$\mathbb{D} \diamond ; (T \diamond c) ; ; \{a, b : T[a, c, b]\}$  i

! 2. Fundamental Proposition of Projections. i

$\vdash \forall T \forall c \forall x \forall y ( (T \diamond c)[x, y] \Leftrightarrow T[x, c, y] )$  i

$T, c$  ,! 1 (Prem) i

$\forall x \forall y ( \{a, b : T[a, c, b]\}[x, y] \Leftrightarrow T[x, c, y] )$  ,! 2 (Pred) i

$\forall x \forall y ( (T \diamond c)[x, y] \Leftrightarrow T[x, c, y] )$  ,! 3 ( $\mathbb{D}I$ : P1,2) i

$\forall T \forall c \forall x \forall y ( (T \diamond c)[x, y] \Leftrightarrow T[x, c, y] )$  ! 4 ( $\forall I$ : 1,3) i

$\square$

! 3. Fundamental Proposition of Projections, First Half. i

$\vdash \forall T \forall c \forall x \forall y ( (T \diamond c)[x, y] \Rightarrow T[x, c, y] )$  i

$T, c, x, y$  ,! 1 (Prem) i

$( (T \diamond c)[x, y] \Leftrightarrow T[x, c, y] )$  ,! 2 ( $\forall E$ : P2) i

$(T \diamond c)[x, y] \Leftrightarrow T[x, c, y]$  ,! 3 ( $(\Rightarrow)E$ : 2) i

$(T \diamond c)[x, y] \Rightarrow T[x, c, y]$  ,! 4 ( $\Leftrightarrow E$ : 3) i

$( (T \diamond c)[x, y] \Rightarrow T[x, c, y] )$  ,! 5 ( $(\Rightarrow)I$ : 4) i

$\forall T \forall c \forall x \forall y ( (T \diamond c)[x, y] \Rightarrow T[x, c, y] )$  ! 6 ( $\forall I$ : 1,5) i

$\square$

! 4. Fundamental Proposition of Projections, Second Half. i

$\vdash \forall T \forall c \forall x \forall y ( T[x, c, y] \Rightarrow (T \diamond c)[x, y] )$  i

$T, c, x, y$  ,! 1 (Prem) i

$( (T \diamond c)[x, y] \Leftrightarrow T[x, c, y] )$  ,! 2 ( $\forall E$ : P2) i

$(T \diamond c)[x, y] \Leftrightarrow T[x, c, y]$  ,! 3 ( $(\Rightarrow)E$ : 2) i

$T[x, c, y] \Rightarrow (T \diamond c)[x, y]$	, ! 4 ( $\Leftrightarrow E$ : 3)	i
$( T[x, c, y] \Rightarrow (T \diamond c)[x, y] )$	, ! 5 ( $(\ ) I$ : 4)	i
$\forall T \forall c \forall x \forall y ( T[x, c, y] \Rightarrow (T \diamond c)[x, y] )$	! 6 ( $\forall I$ : 1, 5)	i
$\square$		
! 5.		
$\vdash \forall T \forall c \forall x \forall y ( ((T \diamond c)^*)[x, y] \Leftrightarrow T[y, c, x] )$		
$T, c, x, y$	, ! 1 (Prem)	i
$( ((T \diamond c)^*)[x, y] \Leftrightarrow (T \diamond c)[y, x] )$	, ! 2 ( $\forall E$ : C3.2)	i
$((T \diamond c)^*)[x, y] \Leftrightarrow (T \diamond c)[y, x]$	, ! 3 ( $(\ ) E$ : 2)	i
$( (T \diamond c)[y, x] \Leftrightarrow T[y, c, x] )$	, ! 4 ( $\forall E$ : P2)	i
$(T \diamond c)[y, x] \Leftrightarrow T[y, c, x]$	, ! 5 ( $(\ ) E$ : 4)	i
$((T \diamond c)^*)[x, y]$	, ! 6 (Prem)	i
$((T \diamond c)^*)[x, y] \Rightarrow (T \diamond c)[y, x]$	, ! 7 ( $\Leftrightarrow E$ : 3)	i
$(T \diamond c)[y, x]$	, ! 8 ( $\Rightarrow E$ : 6, 7)	i
$(T \diamond c)[y, x] \Rightarrow T[y, c, x]$	, ! 9 ( $\Leftrightarrow E$ : 5, 8)	i
$T[y, c, x]$	, ! 10 ( $\Rightarrow E$ : 8, 9)	i
$((T \diamond c)^*)[x, y] \Rightarrow T[y, c, x]$	, ! 11 ( $\Rightarrow I$ : 6, 10)	i
$T[y, c, x]$	, ! 12 (Prem)	i
$T[y, c, x] \Rightarrow (T \diamond c)[y, x]$	, ! 13 ( $\Leftrightarrow E$ : 5)	i
$(T \diamond c)[y, x]$	, ! 14 ( $\Rightarrow E$ : 12, 13)	i
$(T \diamond c)[y, x] \Rightarrow ((T \diamond c)^*)[x, y]$	, ! 15 ( $\Leftrightarrow E$ : 3)	i
$((T \diamond c)^*)[x, y]$	, ! 16 ( $\Rightarrow E$ : 14, 15)	i
$T[y, c, x] \Rightarrow ((T \diamond c)^*)[x, y]$	, ! 17 ( $\Rightarrow I$ : 12, 16)	i
$((T \diamond c)^*)[x, y] \Leftrightarrow T[y, c, x]$	, ! 18 ( $\Leftrightarrow I$ : 11, 17)	i
$( ((T \diamond c)^*)[x, y] \Leftrightarrow T[y, c, x] )$	, ! 19 ( $(\ ) I$ : 18)	i
$\forall T \forall c \forall x \forall y ( ((T \diamond c)^*)[x, y] \Leftrightarrow T[y, c, x] )$	! 20 ( $\forall I$ : 1, 19)	i
$\square$		