

The Solution to the Liar's Paradox

by Andrew Boucher

A solution to the Liar must do two things. First, it should say exactly *which* step in the Liar reasoning - the reasoning which leads to a contradiction - is invalid. Secondly, it should explain *why* this step is invalid.

The Liar reasoning goes like this. Consider the statement:

[L] "Not L is true."

Step 1. Suppose L is true.

Step 2. "Not L is true" and L are the same thing, so by substitution (of equals for equals), "Not L is true" is true.

Step 3. Using the rule "If "S" is true, then S", one concludes that not L is true.

Step 4. Statements 1 and 3 are contradictory, so by reductio ad absurdum, the supposition is not so, i.e. not L is true.

Step 5. From "If S, then "S" is true", it can be inferred that "Not L is true" is true.

Step 6. "Not L is true" and L are the same thing, so by substitution (of equals for equals), L is true.

Step 7. But this is a contradiction.

The flaw in the reasoning is Step 5, because the rule "If S, then "S" is true" is invalid.

Of course, it must now be explained *why* it is flawed. There may be other meanings of "true", but I will consider the one where a proposition is true if the fact which it asserts corresponds with reality. E.g. "George Washington was the first American President" is true because it asserts the fact that George Washington was the first American President, and this corresponds with reality.

On the other hand, "(Classic) Coca-cola is white" is not true, because it asserts a fact which does not correspond with reality.

Let's note that certain referring expressions depend on the reference of other expressions, e.g.

[A] "the referent of [B]"

[B] "George Washington"

So [A] refers to George Washington. This allows for the possibility of viciously referring expressions: expressions which depend on what their own referents would be in order to refer, e.g.

[C] "the referent of [C]"

and

[D] "the referent of [E]"

[E] "the referent of [D]"

None of [C], [D], and [E] refer successfully. As an aside, note that the problem is not self-reference, since

[F] "the referring expression [F]"

refers successfully, to [F]. Viciousness comes when a supposition is made about one's own referent, in order for a referring expression to refer, not from self-reference.

Now let's return to propositions and truths. A proposition is true if and only if the fact which the proposition asserts corresponds with the world (i.e. the fact *holds*). So a proposition is not true in two cases: (a) the fact which it asserts does not correspond with the world; or (b) "the fact which the proposition asserts" does not refer successfully. In case (a) we say that the proposition is false; in (b) that it is empty. Note that one could use "meaningless" or "paradoxical" or "X" for case (b); the name isn't at all important. The important point is coming.

Consider:

[G] "The fact which [H] asserts holds" i.e. "[H] is true"

[H] "Snow is white"

The fact which [H] asserts is that snow is white. Since this holds, [G] is true. But now one can create vicious cases of reference with "the fact", e.g.

[I] "The fact which [I] asserts holds" i.e. "[I] is true".

The referring expression in this proposition, "the fact which [I] asserts," is vicious, because its reference depends on what it itself would refer to. So [I] does not assert any fact; it is empty. Similarly,

[J] "The fact which [J] asserts does not hold," i.e. "[J] is false,"

is empty. Now "not true" is "false or empty," so the Liar is equivalent to:

[K] "The fact which [K] asserts does not hold, or [K] does not assert a fact," i.e. "[K] is not true."

Now "the fact which [K] asserts" is vicious, because its reference depends on what fact it itself would assert. So, [K] does not assert a fact. To repeat:

[K] does not assert a fact. (Call this [M]).

Thus:

The fact which [K] asserts does not hold, or [K] does not assert a fact, i.e. [K] is not true. (Call this [N]).

But [N] is just the assertion [K]. So when S is [K], we have both S and "S" is not true. To repeat, we have both that

The fact which [K] asserts does not hold, or [K] does not assert a fact (S)

and

[K] is not true ("S" is not true).

Now normally we would also be able to conclude that [K] is true, using the rule "If S, then "S" is true", but hopefully it is clear why this rule does not apply. It applies in the case when S asserts a fact, but here there is none. Indeed, the reason why the Liar is special is that it is a disjunction, where one disjunct is empty, and the other disjunct, [M], is true. The over-all disjunction is still empty, because emptiness is contagious. And this means that the over-all disjunction is not true, even if one disjunct is true. (Emptiness takes precedence over truth in a disjunction.)

Finally, it should be emphasized that there is no change of logic here. Logic stays at it is. If B, then A or B. For example, if [K] does not assert a fact, then the fact which [K] asserts does not hold, or [K] does not assert a fact. What one doesn't have is the truth-table rule, "If B is true, then (A or B) is true." This holds when statements are normal, where the rule "If S, then "S is true"" applies, but not necessarily in the special cases where it does not.

BIBLIOGRAPHY

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